Assumed knowledge for Quantitative Biology

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1 Introduction

The following is a list of assumed knowledge for QB. If you remember about all of these from your school work then you are in a very good position to tackle this term's work. If you don't, you have some revision to do – we can talk about any gaps in our first supervision. A lot of the first term's work concerns these matters and is basically revision in context, adding the key idea of the **differential equation**, so you will have a chance to practice any skills that you think are rusty over the next few weeks.

1.1 Algebra

- Basic algebra
  - expanding brackets out quickly and accurately
  - binomial expansion for integral values of the power
  - "difference of two squares"
  - formula for solving quadratic equations

$$ax^2 + bx + c = 0 \Longleftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

- Partial fractions
- The laws of indicies

$$x^a \cdot x^b = x^{a+b} \tag{2}$$
$$\frac{x^a}{x^b} = x^{a-b} \tag{3}$$
$$(x^a)^b = x^{ab} \tag{4}$$

- The definition of a logarithm

$$x^a = b \Longleftrightarrow log_x(b) = a \tag{5}$$

  and knowledge that in Cambridge if you see $\log x$ (no subscript) this is assumed to mean "log to the base e" or what your school teachers might have called `ln`.

- The laws of logarithms

$$\log(ab) = \log(a) + \log(b) \tag{6}$$
$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b) \tag{7}$$
$$\log\left(a^b\right) = b\log(a) \tag{8}$$

1.2 Graphs

- Basic polynomial forms

$$y = ax + b \tag{9}$$
$$y = ax^2 + bx + c \tag{10}$$
$$y = (x-a)(x-b)(x-c) \tag{11}$$

  for different values of a, b, c

- Basic trigonometric graphs

$$y = \sin(x) \tag{12}$$
$$y = \cos(x) \tag{13}$$

- Graphs of e^x and $\log(x)$
- Transformations – if you know the shape of $y = f(x)$ what are the shapes of
  - $y = f(x) + \kappa$ - translation up/down y axis
  - $y = \kappa f(x)$ - scaling up/down y axis (flips in x axis if $\kappa < 0$)
  - $y = f(\kappa x)$ - scaling up/down x axis (flips in y axis if $\kappa < 0$)
  - $y = f(x + \kappa)$ - translation up/down x axis

  where $\kappa > 0$ or $\kappa < 0$.
- Idea of an asymptote (vertical and horizontal only – no need for oblique asymptotes)
- More complicated forms involving division and generally requiring calculus to sketch - for example the graph of

$$y = \frac{\kappa x}{b + cx} \tag{14}$$

1.3 Differentiation

- Basic results

$$y = x^n \iff \frac{dy}{dx} = nx^{n-1} \tag{15}$$

$$y = \sin(nx) \iff \frac{dy}{dx} = n\cos(nx) \tag{16}$$

$$y = \cos(nx) \iff \frac{dy}{dx} = -n\sin(nx) \tag{17}$$

$$y = e^{nx} \iff \frac{dy}{dx} = ne^{nx} \tag{18}$$

$$y = \log(nx) \iff \frac{dy}{dx} = \frac{1}{x} \tag{19}$$

- Product rule

$$y(x) = u(x)v(x) \implies \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \tag{20}$$

- Quotient rule

$$y(x) = \frac{u(x)}{v(x)} \implies \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \tag{21}$$

- "Chain" or "function of a function" rule

$$y = u(v(x)) \implies \frac{dy}{dx} = \frac{du}{dv}\frac{dv}{dx} \tag{22}$$

  For example to differentiate $y = e^{x^3}$ set $y = u = e^v$ and $v = x^3$

$$\frac{dy}{dx} = e^v \cdot 3x^2 = 3x^2 e^{x^3} \tag{23}$$

1.4 Integration

- Reverse of the basic results (where C is a constant of integration)

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \tag{24}$$

$$\int \frac{1}{x} dx = \log(x) + C \tag{25}$$

$$\int \sin(nx) dx = -\frac{1}{n}\cos(nx) + C \tag{26}$$

$$\int \cos(nx) dx = \frac{1}{n}\sin(nx) + C \tag{27}$$

$$\int e^{nx} dx = \frac{1}{n} e^{nx} + C \tag{28}$$

- Integration by substitution
- Integration by parts

1.5 Applications of Calculus

- Turning point of a curve when $\frac{dy}{dx} = 0$ and discriminating between maxima and minima based on sign of $\frac{d^2y}{dx^2}$
- Inflection point of a curve when $\frac{d^2y}{dx^2} = 0$
- Area under a curve by definite integration (ie. integrating between two limits)
- **For those of you with Further Maths:** revise any work you have done on "differential equations" as it will be quite useful this term.